GRID-SEARCH EVENT LOCATION WITH NON-GAUSSIAN ERROR DISTRIBUTIONS

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GMEL = Grid-search Multiple-Event Location

- Solves

$$d_{ij} = T_j(\mathbf{x}_i) + t_i + c_j + e_{ij} \quad \text{(not all } i, j \text{ pairs)}$$

where

d_{ij} = arrival time for event i and station/phase combination j

$\mathbf{x}_i$ = hypocenter of event i

t_i = origin time of event i

c_j = travel-time correction for station/phase j

T_j = model-based travel-time function for station/phase j

e_{ij} = observational (pick) error in d_{ij}

- Multiple-event location: solve jointly for the $(\mathbf{x}_i, t_i)$ and c_j

- Single-event location: fix the c_j

GMEL (cont'd)

- GMEL assumes the pick errors have a <u>generalized Gaussian</u> probability distribution:

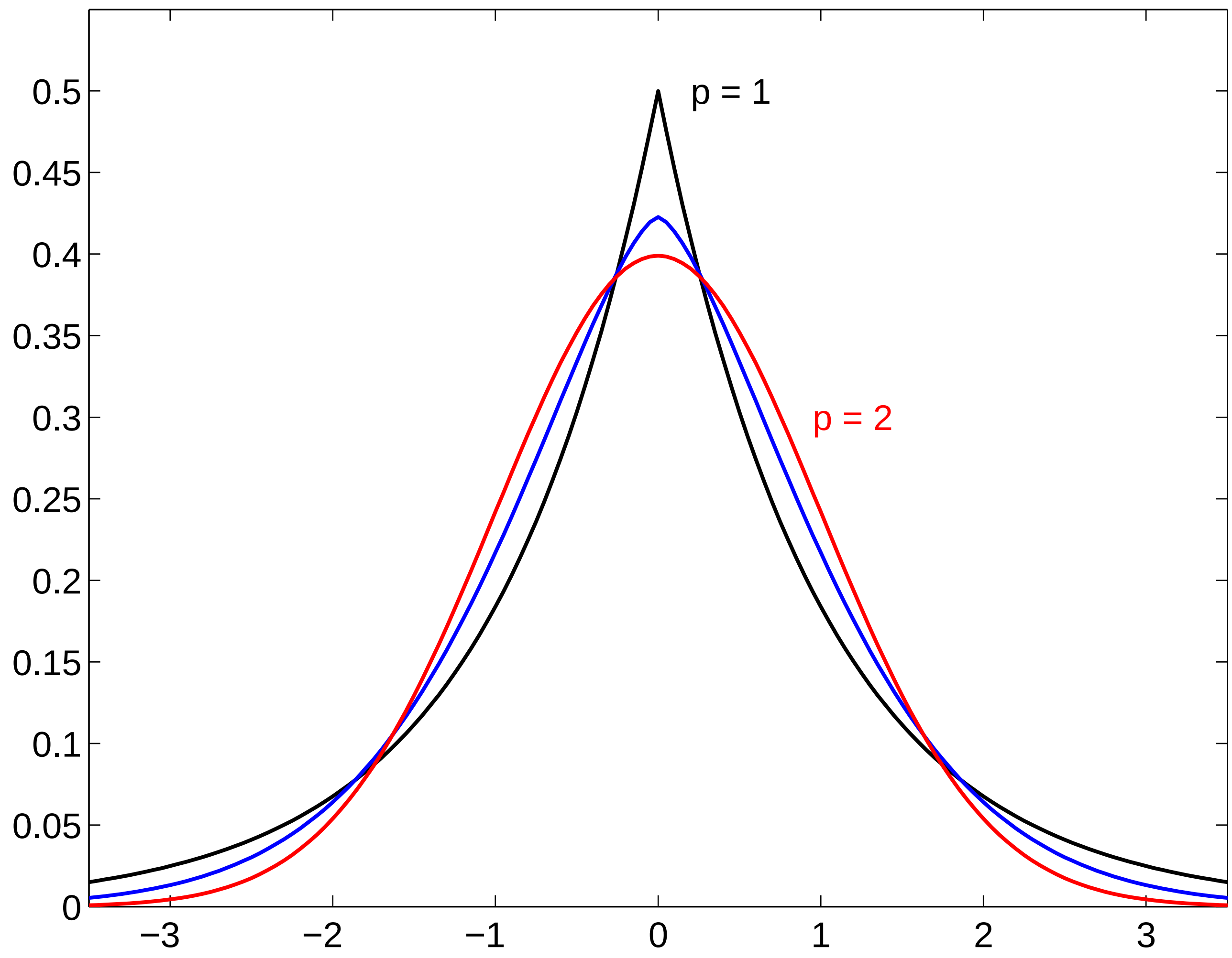
$$f(e_{ij}) = \text{const} \times \exp\left\{-\frac{1}{p}\left(\frac{e_{ij}}{\sigma_{ij}}\right)^p\right\}$$

where

$$\sigma_{ij} = \text{standard error of } e_{ij}$$

- Cases considered in this talk:
  - $p = 2$: Gaussian error distribution
  - $p = 1$: Laplace error distribution
  - $p = 1.5$

Generalized Gaussian Probability Distribution



GMEL (cont'd)

- GMEL maximizes the likelihood function given by

$$-\log L = \text{const} + \sum_{ij} \log \sigma_{ij} + \frac{1}{p} \sum_{ij} \frac{1}{(\sigma_{ij})^p} \left| d_{ij} - T_j(\mathbf{x}_i) - t_i - c_j \right|^p$$

- The event locations $(\mathbf{x}_i, t_i)$ and travel-time corrections c_j minimize a weighted L_p norm of the data residuals.

- GMEL uses
  - Adaptive grid-search to find $\mathbf{x}_i$
  - Regula falsi root-finding to find t_i and c_j
  - Travel-time tables for T_j (not on-the-fly calculation)

Tests With the IWREF Date Set

- Most tests used the ISC defining phases (for 154 of 156 events)

34,614 P	3567 Pn	262 Pb	275 Pg
39 S	10 Sn	0 Sb	1 Sg

- Some tests used an additional 9864 non-defining phases (direct arrivals, no depth phases)

- Assumed nominal accuracy (σ_{ij})
  - P phases: 1.0 sec
  - S phases: 2.0 sec

- Phase re-naming was done, e.g.

PN $\mapsto$ Pn P* $\mapsto$ Pb PKP2 $\mapsto$ PKPab

- AK135 travel-time tables were used for the forward model. Composite first-arrival tables were used for

  Pg/Pb   Pn/P/Pdiff   PKP/PKPxx

- Location constraints (for speed):
  - Epicenters within 500 km of IWREF locations
  - Depths between 0 and 250 km

Gaussian vs. non-Gaussian Solutions

— Epicenter Mislocation —

	Case	*mean*	*median*	*90%-ile*	*max*	
	L2	13.3	10.6	25.3	84.4	
	L1.5	9.7	7.9	18.7	45.1	
→	L1	9.4	7.7	17.8	45.7	←

— Depth Mislocation —

	Case	*mean*	*median*	*90%-ile*	*max*	
	L2	15.7	6.4	32.4	237.8	
	L1.5	13.1	6.9	29.8	191.8	
→	L1	12.9	6.0	28.3	198.6	←

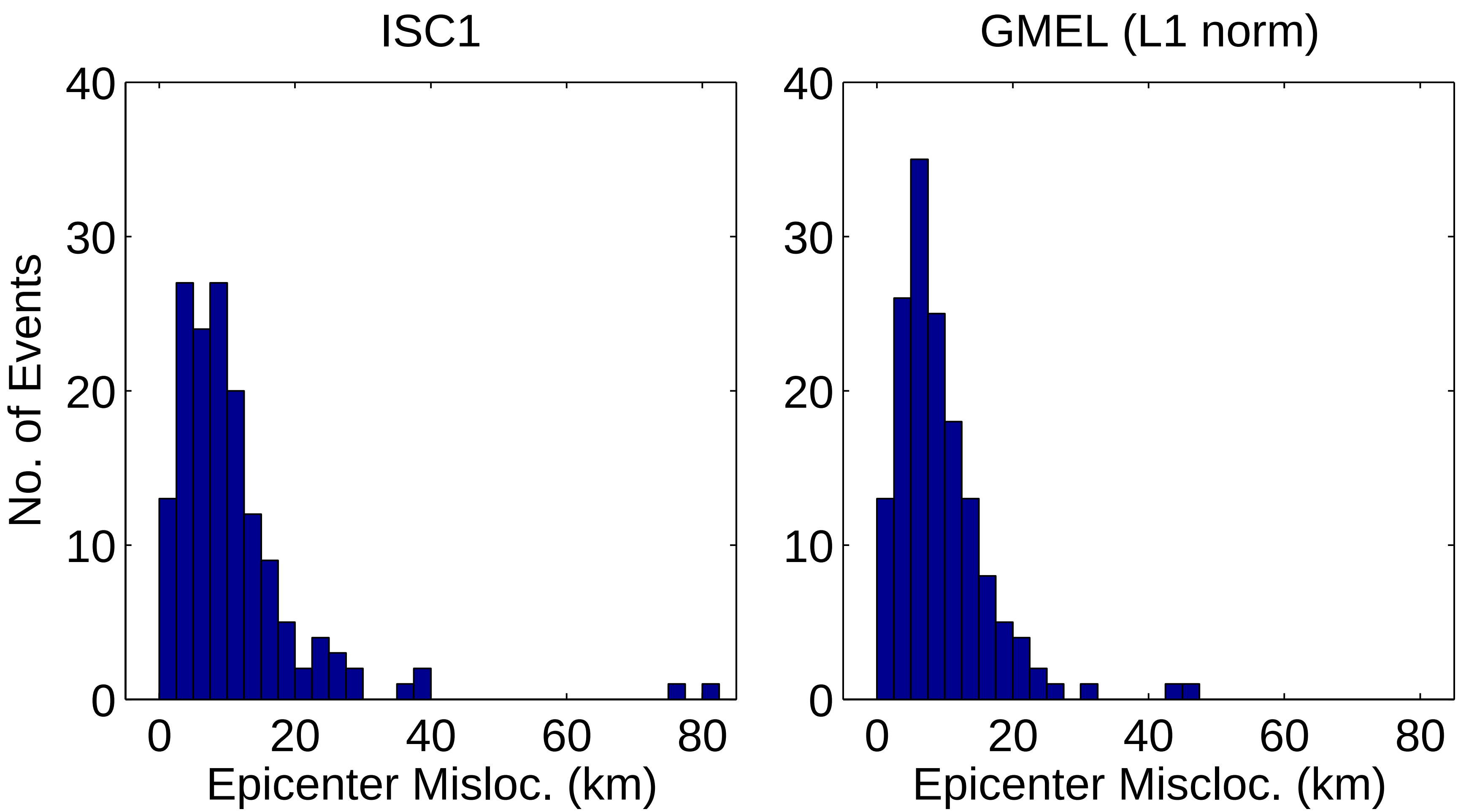
ISC1 vs. GMEL Solutions

- Based on 153 events (one problematic ISC solution omitted)

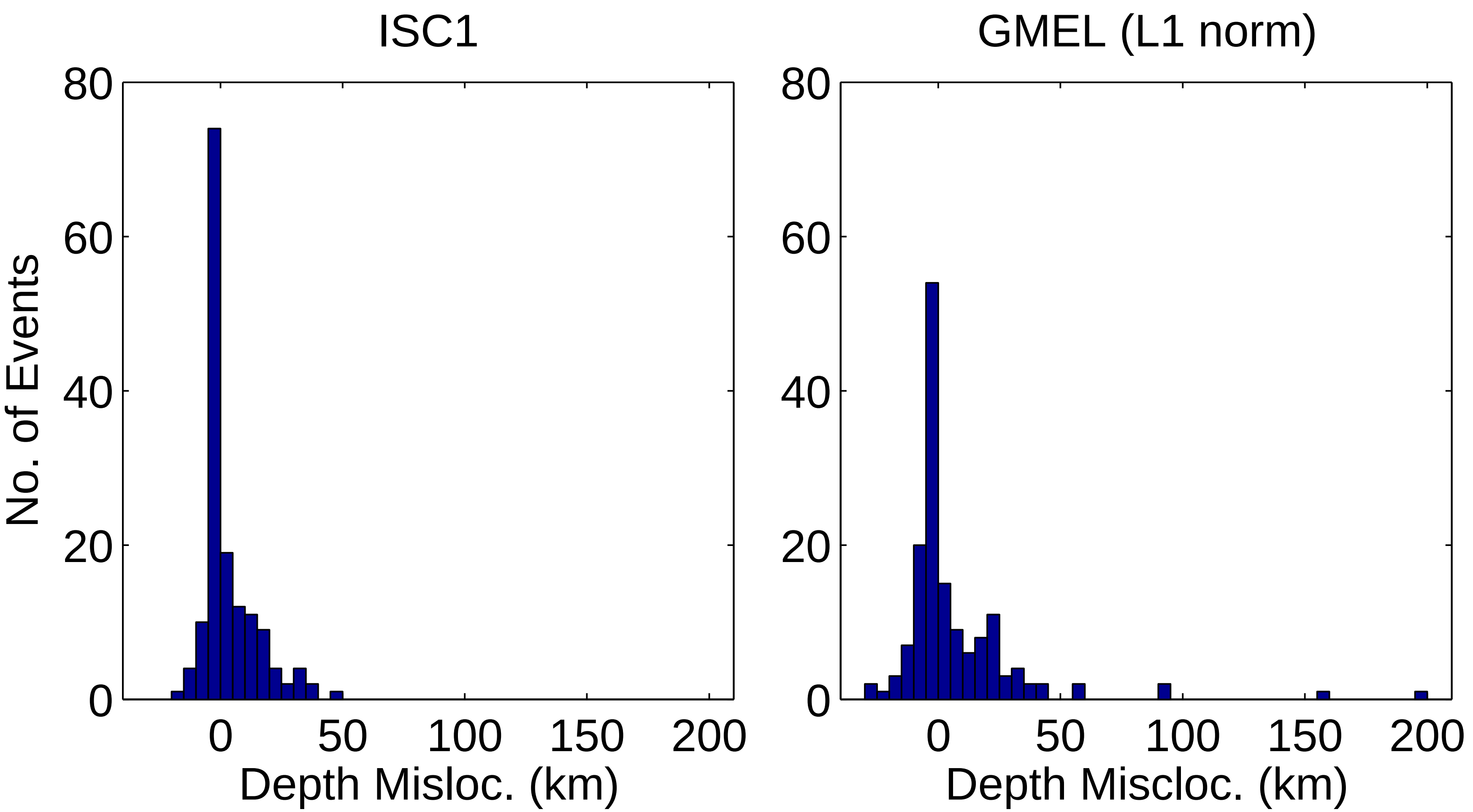
	— Epicenter Mislocation —			
Solution	*mean*	*median*	*90%-ile*	*max*
ISC1	11.0	8.7	20.1	80.8
GMEL (L2 norm)	13.3	10.5	25.3	84.4
GMEL (L1 norm)	9.4	7.7	17.3	45.7

	— Depth Mislocation —			
Solution	*mean*	*median*	*90%-ile*	*max*
ISC1	6.8	2.7	19.6	48.7
GMEL (L2 norm)	15.8	6.5	32.4	237.8
GMEL (L1 norm)	12.9	6.0	28.3	198.6

Epicenter Mislocations



Depth Mislocations



Weighting by Data Quality

- Set the data standard errors (σ_{ij}) as

Quality	P phases	S phases
Impulsive	0.5 sec	1.0 sec
Emergent	1.5	3.0
Unspecified	1.0	2.0

Quality-Weighting Solutions

Case	— Epicenter Mislocation —			
Case	*mean*	*median*	*90%-ile*	*max*
L1	9.4	7.7	17.8	45.7
L1 qual-weight	9.6	7.8	18.0	46.1
L2	13.3	10.6	25.3	84.4
L2 qual-weight	14.0	10.0	31.8	84.9

Case	— Depth Mislocation —			
Case	*mean*	*median*	*90%-ile*	*max*
L1	12.9	6.0	28.3	198.6
L1 qual-weight	13.0	6.0	27.4	202.1
L2	15.7	6.4	32.4	237.8
L2 qual-weight	16.0	8.3	35.1	194.4

Data Re-Weighting

- Likelihood function:

$$-\log L = \text{const} + \sum_{ij} \log \sigma_{ij} + \frac{1}{p} \sum_{ij} \frac{1}{(\sigma_{ij})^p} \left| d_{ij} - T_j(\mathbf{x}_i) - t_i - c_j \right|^p$$

- GMEL can let the σ_{ij} be unknowns, if constrained.
  - Station/phase re-weighting:

$$\sigma_{ij} = \sigma_j \, \nu_{ij}$$

  - Solve for the σ_j subject to

$$0.5 \leq \sigma_j \leq 2$$

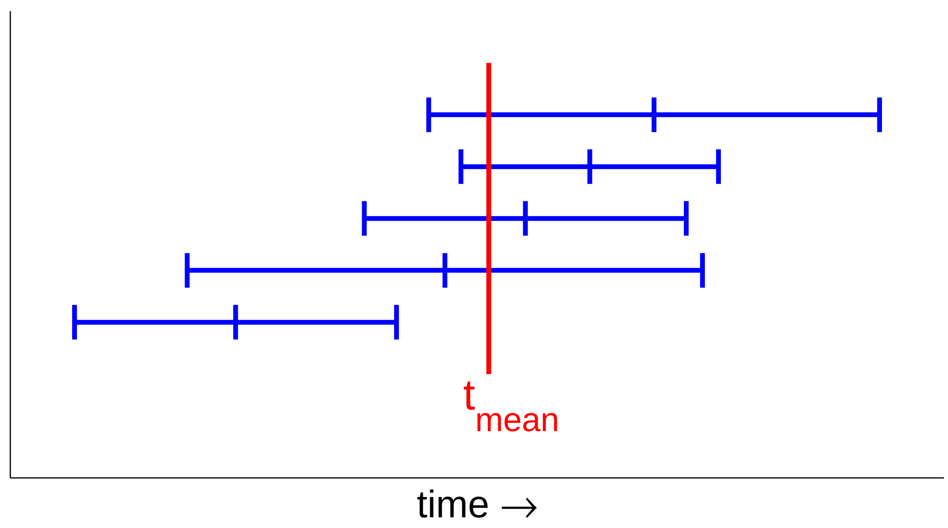
Association (Outlier Rejection)

- Given n arrival times from an event, t_j, set

$$w_j = 3.7\,\sigma_j + \left(\frac{\theta_{reso}}{\Delta}\right) T_j^{cal} + 10\,t_{reso}$$

$$t_{\text{mean}} = \frac{1}{\sum_{j=1}^{n} {w_j}^{-1}} \sum_{j=1}^{n} {w_j}^{-1}\, t_j$$

- Associate (<u>accept</u>) arrival j if: $\quad t_j - w_j \leq t_{\text{mean}} \leq t_j + w_j$



Re-Weighting/Association Solutions (L1 cases)

	— Epicenter Mislocation —			
Case	*mean*	*median*	*90%-ile*	*max*
L1	9.4	7.7	17.8	45.7
L1 re-weight	9.8	8.0	17.5	51.7
L1 associate	9.5	7.8	18.0	45.3

	— Depth Mislocation —			
Case	*mean*	*median*	*90%-ile*	*max*
L1	12.9	6.0	28.3	198.6
L1 re-weight	14.0	6.3	26.2	203.5
L1 associate	12.8	5.9	27.6	197.9

Re-Weighting/Association Solutions (L2 cases)

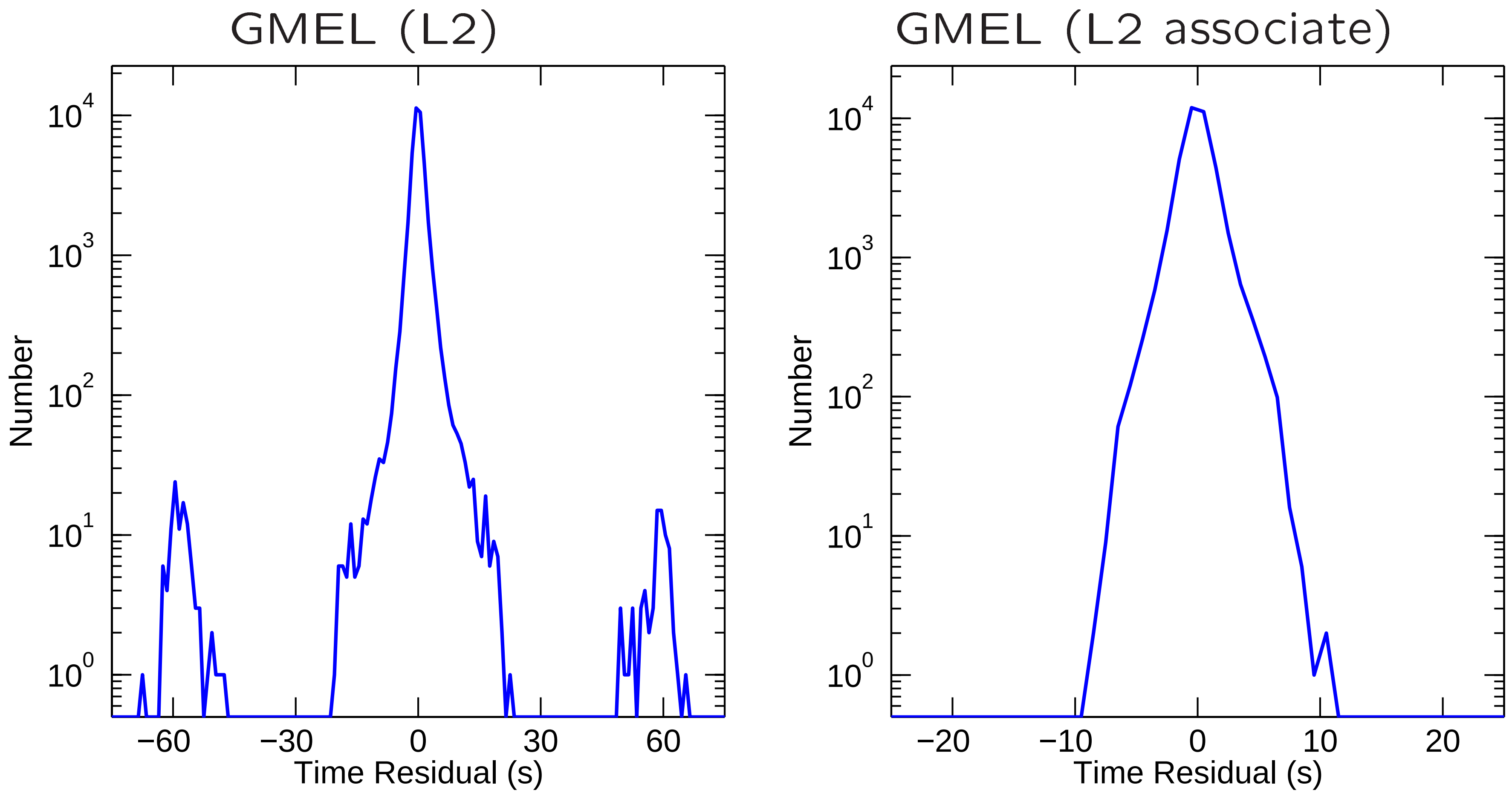
— Epicenter Mislocation —

Case	*mean*	*median*	*90%-ile*	*max*
L1	9.4	7.7	17.8	45.7
L2	13.3	10.6	25.3	84.4
L2 re-weight	10.7	8.6	20.3	94.0
L2 associate	10.0	8.2	18.7	41.8

— Depth Mislocation —

Case	*mean*	*median*	*90%-ile*	*max*
L1	12.9	6.0	28.3	198.6
L2	15.7	6.4	32.4	237.8
L2 re-weight	13.9	6.4	31.3	194.0
L2 associate	12.5	5.9	28.8	182.3

Residuals After Re-location (ISC defining phases)



Residual Statistics (ISC defining phases)

- Weighted Residual Norm

$$\text{W.R.N.} = \left(\frac{\sum_{ij} (r_{ij}/\sigma_{ij})^p}{\sum_{ij} (1/\sigma_{ij})^p} \right)^{\frac{1}{p}}$$

Case	*ndef*	*W.R.N.*
ISC1	38,766	4.7
GMEL L2	38,768	4.5
GMEL L2 re-weight	38,768	1.9
GMEL L2 associate	37,985	1.5
GMEL L1	38,768	1.5
GMEL L1 re-weight	38,768	1.1
GMEL L1 associate	38,273	1.1

Multiple-Event Location

- Likelihood function:

$$-\log L = \text{const} + \sum_{ij} \log \sigma_{ij} + \frac{1}{p} \sum_{ij} \frac{1}{(\sigma_{ij})^p} \left| d_{ij} - T_j(\mathbf{x}_i) - t_i - c_j \right|^p$$

- Solve for the travel-time correction for each station/phase, c_j, subject to

$$-3 \text{ sec} \leq c_j \leq 3 \text{ sec}$$

Multiple-Event Solutions
(L1 cases)

	— Epicenter Mislocation —			
Case	*mean*	*median*	*90%-ile*	*max*
L1 SEL	9.4	7.7	17.8	45.7
L1 MEL	9.3	7.8	17.8	42.8
L1 MEL re-weight	9.4	7.6	18.1	42.6
L1 MEL associate	10.0	9.1	17.8	37.4

	— Depth Mislocation —			
Case	*mean*	*median*	*90%-ile*	*max*
L1 SEL	12.9	6.0	28.3	198.6
L1 MEL	13.1	6.5	27.6	152.2
L1 MEL re-weight	12.6	6.3	30.2	154.0
L1 MEL associate	14.4	8.1	33.3	163.8

Multiple-Event Solutions
(L2 cases)

	— Epicenter Mislocation —			
Case	*mean*	*median*	*90%-ile*	*max*
L1 SEL	9.4	7.7	17.8	45.7
L2 SEL	13.3	10.6	25.3	84.4
L2 MEL	15.8	12.6	30.1	97.1
L2 MEL re-weight	11.6	9.1	20.1	101.1
L2 MEL associate	11.0	9.2	19.0	41.2

	— Depth Mislocation —			
Case	*mean*	*median*	*90%-ile*	*max*
L1 SEL	12.9	6.0	28.3	198.6
L2 SEL	15.7	6.4	32.4	237.8
L2 MEL	16.4	7.5	39.0	238.4
L2 MEL re-weight	14.5	8.8	28.9	144.8
L2 MEL associate	14.6	7.8	32.5	144.6

All-Phases Solutions
(L1 cases)

	— Epicenter Mislocation —			
Case	*mean*	*median*	*90%-ile*	*max*
L1 def-only	9.4	7.7	17.8	45.7
L1 all	115.7	14.5	366.2	497.9
L1 all re-weight	116.5	14.6	375.8	497.9
L1 all associate	21.0	9.3	60.9	155.9

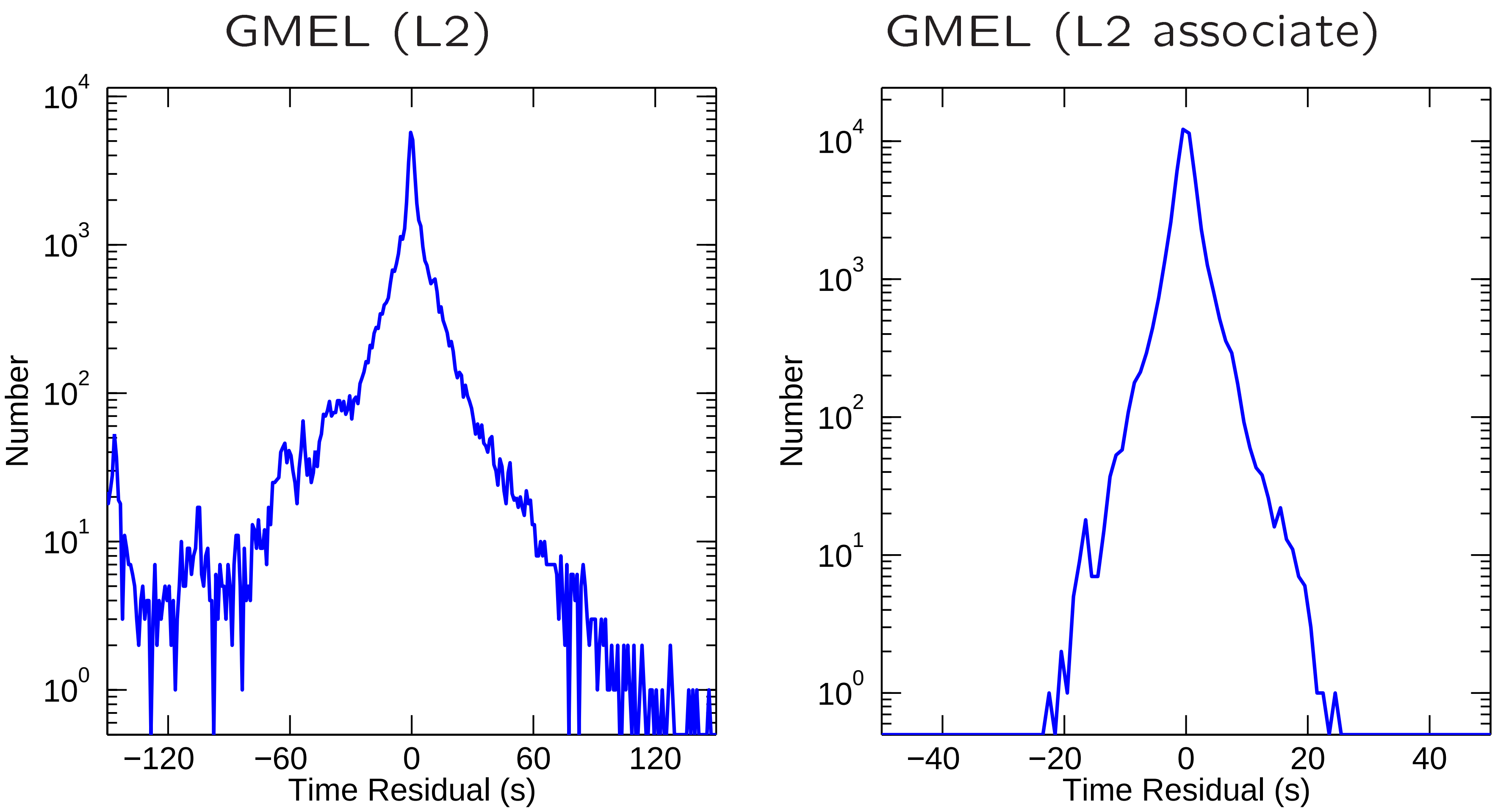
	— Depth Mislocation —			
Case	*mean*	*median*	*90%-ile*	*max*
L1 def-only	12.9	6.0	28.3	198.6
L1 all	27.9	9.1	62.0	250.0
L1 all re-weight	29.3	8.8	88.5	230.0
L1 all associate	12.3	7.0	26.0	132.8

All-Phases Solutions
(L2 cases)

	— Epicenter Mislocation —			
Case	*mean*	*median*	*90%-ile*	*max*
L1 def-only	9.4	7.7	17.8	45.7
L2 def-only	13.3	10.6	25.3	84.4
L2 all	125.7	41.8	370.4	500.0
L2 all re-weight	121.2	28.5	371.6	500.0
L2 all associate	12.8	8.7	30.1	82.0

	— Depth Mislocation —			
Case	*mean*	*median*	*90%-ile*	*max*
L1 def-only	12.9	6.0	28.3	198.6
L2 def-only	15.7	6.4	32.4	237.8
L2 all	40.5	10.4	169.5	250.0
L2 all re-weight	29.9	10.0	84.0	249.4
L2 all associate	12.3	6.5	26.0	146.3

Residuals After Re-location
(all direct-depth P and S phases)



Residual Statistics
(all direct P and S phases)

Case	*ndef*	*W.R.N.*
L2 def-only	38,768	4.5
L2 all	48,632	652.4
L2 all associate	47,171	2.2
L1 def-only	38,768	1.5
L1 all	48,632	17.8
L1 all associate	47,829	2.0

Conclusions

- In our tests with the IWREF data set, automated location procedures, applied to direct P and S arrivals, improved ISC epicenters but not depths.

- The improvement resulted from
  - L1 norm minimization (non-Gaussian error model)
  - simple outlier rejection (association)
  - re-weighting data by station/phases

- More sophisticated procedures (than GMEL uses) are needed to improve automated location with raw ISC picks.

- More investigation is needed to determine if multiple-event location techniques can further improve automatic location procedures.

Acknowledgments

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